2024-2025
Cosmology (4 crédits = 48.5 h)

1. **The observables (18 h – 9 lectures of 2 hours) [O1 to O9]**

   1.1. Objects and structures in the universe (4 h) [O1, O2]
   Objects: Galaxies (individual and collectives properties) + QSO + CGM/IGM
   1.2. Hierarchical structures (8 h) [O4, O5, O7, O9]
   Groups of galaxies (including the Local Group) - Clusters of galaxies - Superclusters - Large scales structures (Voids – Filaments – redshift space)
   1.3. The pillars of cosmology (6 h)
   The Hubble Law (expansion) + Distant SN (acceleration of the expansion) [O3]
   The Cosmic Microwave Background [O6]
   Primordial nucleo-synthesis + Current issues [O8]

2. **Friedmann-Lemaître models (20 hours – 10 lectures of 2 hours) [M1 to M10]**

   2.1. Introduction to general relativity (RG) (4h) [M1, M2]
   Tensor calculus - Curved spacetime – Energy-momentum tensors – Equivalence, RG and covariance principles – Formal Einstein equations.
   2.2. Friedmann–Lemaître–Robertson–Walker (FLRW) (5h) [M3, M4, M5/2]
   Einstein Eq. in the Friedman-Lemaître model for the Robertson-Walker metric: Calculus of the Christoffel symbols – Ricci tensors – Scalar Curvature – Einstein and energy-momentum tensors – Einstein equations.
   2.3. Homogeneous universe (5h) [M5/2, M6, M7]
   Scale factor equation – Densities - First and second equation of cosmologies – Hubble law – Deceleration parameter – Lookback time - Models without and with cosmological constant – Einstein-de Sitter models.
   2.4. Properties of FLRW models (5h) [M8, M9, M10/2]
   Density parameters – Galaxy luminosity function – Universe Mass-to-Light ratio - Evolution parameters – Radial trajectory of photons – Cosmic time - Proper and comoving distances – Redshift-apparent magnitude relation – deceleration parameter from distant supernovae.
   2.5. Introduction to Inflationary models - adiabatic and isothermal density fluctuations. (1h) [M10/2]

3. **Project (10.5 hours : 6 x 1h45') [P1 to P6]**

   Students will choose one item of the project :
   Item 1 : Friedmann equations in ΛCDM and dark-energy cosmologies
   Item 2 : Number counts of galaxy clusters in dark-energy cosmologies
   About 50% of students will do item 1, and 50% item 2.

HCM : 17 + HTD : 17 + HTP : 6 → EDT : 17*1.5+17+6 = 48.5 HEqTD

CA : Christophe Adami (12 h eq. TD = 6 HCM + 3 HTD)
PhA : Philippe Amram (26 h eq. TD = 11 HCM + 9.5 HTD)
CS: Carlo Schimd (10.5 h eq. TD = 6 HTP + 4.5 HTD)

Calendar

Month	Jan	Jan	Jan	Jan	Feb	Feb	Feb	Feb	Mar	Mar	Mar	Mar
Week	1	2	3	4	5	6	7	8	9	10	11	12
1. Observables (CA & PhA)	O1/CA	O2/CA	O3/PhA	O4/CA	O5/CA	O6/PhA	O7/CA	tmp	O8/PhA	tmp	O9/CA	tmp
2. Models FLRW (PhA)	M1	M2	M3	M4	M5	M6	M7	M8	tmp	M9	tmp	M10
3. Project (CS)				P1	P2	P3	P4	P5		P6		
Christophe Adami	CA		Philippe Amram	PhA			Carlo Schimd	CS			Exam in April	

Detailed description of the project :

The first lecture will introduce the project and the numerical techniques employed to solve the problems (interpolation, integration, solution of ODE).

By the second lecture, students will train on numerical techniques using the language of their choice (Python, Mathematica, Matlab, Fortran, C, C++, ...), eventually installing the appropriate libraries (e.g. astropy, cosmopy, ...), and train with TopCat or equivalent software. The second lecture will be done for separate groups, illustrating the details of the project.

The next lectures will be open to both groups, dedicated to programming and problem solving.

Item 1 : Friedmann equations in ΛCDM and dark-energy cosmologies

- Solve numerically the Friedmann equations [*] for several values of cosmological parameters not admitting analytical solutions, considering a model with matter (m), radiation (r), cosmological constant (Λ), and general spatial curvature (when known, the analytical solutions will be compared to numerical ones).
- Plot of the scale factor a(t), Hubble parameter H(t), expansion velocity da/dt as function of time or redshift for the different models. Optional: consider dark energy models with constant or time-varying equation-of-state (CPL parametrization), then models with non-analytical H(z) (tabulated ascii file).
- Derive the comoving radial distance, angular diameter distance, luminosity distance, look-back time, and volume for the different models; plots as function of redshift. Deduce the relative deviations w.r.t. ΛCDM.

- Plot the iso-contours in the $(\Omega m, \Omega\Lambda)$ plane of the Hubble parameter $H(z,\Omega m,\Omega\Lambda)$, comoving radial distance $\chi(z,\Omega m,\Omega\Lambda)$, and look-back time $t(z,\Omega m,\Omega\Lambda) - t0$ at fixed redshift (z = 0.1, 1, 10, 100, 1000), for ΛCDM model with fixed value of Ωr.
- Optional: repeat for alternative dark energy models.
- Plot the angle (in arcsec, arcmin, or deg) of sources of physical length typical of galaxy, galaxy groups, and galaxy clusters, as function of redshift, for several cosmologies.
- Compare the results.

[*] numerical methods: integration of ODE, integration.

Item 2 : Number counts of galaxy clusters in dark-energy cosmologies

- Starting from analytic expressions of the Hubble parameter H(z) in ΛCDM and XCDM cosmologies, in which dark energy is modeled by constant or time-varying equation-of-state (e.g. CPL parametrization), compute [**] the comoving radial distance, angular diameter distance, luminosity distance, look-back time, and volume.
- Optional: consider dark energy models with non-analytical H(z) (tabulated ascii file). Deduce the relative deviations w.r.t. ΛCDM.
- Deduce the flux fX (erg/s/cm2) for typical absolute X-ray luminosity LX of galaxy clusters (see bibliography). Assuming a specific mass-luminosity relation, determine the mass range and redshift range observable in the XMM-XXL and eROSITA surveys.
- For a survey of 10,000 deg2 over the redshift range $0 < z < 1$, integrate a Schechter mass function appropriate for galaxy clusters to estimate the proper number density n(z). Deduce the number counts of clusters N(>M) having mass larger than M = 10^13, 10^14, 10^15 h^-1M⊙ in a redshift band dz and in the full redshift range.
- Extract volume-limited subsample in three redshift ranges of equal width Δz and equal magnitude ΔM from "The new X-Class catalogue" (see https://cdsarc.cds.unistra.fr/ftp/J/A+A/652/A12/).
- Check the consistency of the different subsamples by supposing a Poisson statistics, for the ΛCDM cosmological model and an alternative model of your choice.

[**] numerical methods: interpolation, integration.